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On α-compact sets in ideal topological spaces

Eman Almuhur [1,*], Mona Khandaqji [1], Mona Bin-Asfour [2], Manal Al-Labadi [3], Wasim Audeh [3]

[1]Department of Mathematics, Faculty of Science, Applied Science Private University, Amman, Jordan
[2]Department of Mathematics and Statistics, Imam Mohammad Ibn Saud Islamic University, Riyadh, Saudi Arabia
[3]Department of Mathematics, Faculty of Arts and Sciences, University of Petra, Amman, Jordan

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ABSTRACT

This paper examines the properties of α-continuous functions modulo (J, I) that map a countably α-compact ideal space (X, τ, J) to an ideal space (Y, σ, I), where Y is an α-closed subset of the Cartesian product (X × Y, τ × σ, J × I). It is shown that if (X, τ, J) has a weight of at least $\aleph_0$, it is the α-continuous image of a closed subspace of the cube $D^{\{\aleph_0\}}$. Additionally, an α-continuous function f: (X, τ, J) → (Y, σ, I), where Y is countably α-compact, can be extended under specific conditions. The concept of α-pseudocompactness is introduced in an ideal topological space (X, τ, J), and it is established that countably α-pseudocompactness is neither finitely multiplicative nor hereditary with respect to α-closed sets. Furthermore, it is proven that an α-continuous function modulo ideals mapping a Tychonoff ideal space to a countably α-pseudocompact space is perfect, and the Tychonoff space itself is countably α-pseudocompact.

1. Introduction

Many important properties—such as resolvability, compactness, connectedness, separation axioms, and the decomposition of continuity—began to be generalized in the context of ideal topological spaces, starting with the work of Janković and Hamlett (1990) and later expanded upon by Almuhur and Bin-Asfour (2022). Kuratowski (1966) was the first to propose ideal topological spaces, and Vaidyanathaswamy (1944) developed this concept. Given a topology τ on X, an ideal I is a non-empty collection of subsets of a non-empty set X that satisfies the hereditary property and the finite additivity property. Many mathematicians worldwide are actively studying generalized open sets because of their significance in topology. Many mathematicians worldwide are actively studying generalized open sets because of their significance in topology.

Studying variously modified versions of separation axioms, continuity, and other concepts using extended open sets is a major issue in topology and analysis. Typically, the most well-known and influential concepts are those of generalized closed (g-closed) subsets of a topological space (Levine, 1970) and α-open sets (Njåstad, 1965).

Subsequently, a sizable mathematical community has focused on generalizing many topological concepts through the usage of $\alpha-$open sets and generalized closed sets. Dunham (1982) defined a new topological space (X,τ^*) by using g-closed subsets of X to define a new closure operator. His method involved transferring regularity conditions from a topological space (X,τ) to separation conditions in the new topological space (X,τ^*).

Since Császár (2007) presented the idea of generalized topological spaces in the 20th century, many mathematicians from all over the world have studied it. As a result, mathematicians attempted to incorporate topological concepts into this discipline by altering their approach.

Consequently, mathematicians changed their strategy and tried to apply topological ideas to this field. A group of subsets of X closed under an arbitrary union is called a GT μ (Levine, 1963).

Throughout this paper, X and Y denote the ideal spaces (X,τ,J) and (Y,σ,I). Typically, a subset A of (X,μ) is $\mu-$open *if and only if* $\forall x\in A, \exists N$ a $\mu-$open neighborhood of x such that $N\subseteq A$. $\mu-$open sets are elements of μ, and if $F\subseteq\mu$ is $\mu-$closed if $X-F$ is $\mu-$open.

Definition 1.1: In the ideal space X, if U is an open subset of X, then:

* Corresponding Author.
Email Address: e_almuhur@asu.edu.jo (E. Almuhur)
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Corresponding author's ORCID profile:
https://orcid.org/0000-0002-9837-4179

i. If $U \subseteq Int\,(Cl\,(Int\,(U)))$, then U is α −open.
ii. U is an α −closed if $Cl(Int(Cl(U))) \subset U$.
iii. G is g −closed (Levine, 1970) if $Cl(G) \subset O(G) \subset \tau$.

Njåstad (1965), building on earlier concepts introduced by Vaidyanathaswamy (1944), showed that every open set is α −open. Similarly, Dunham (1997) demonstrated that the property of g −closedness is closed under arbitrary union, extending the earlier work of Levine (1970).

Definition 1.2:

(i) The ideal space X is $T_{\frac{1}{2}}$ if and only if for each $x \in X$, $\{a\}$ is either closed or open if and only if:

$$\{a\} = \bigcap\{A: a \in A\,, A\ is\ open\ in\ X\}$$

or $\{a\} = \bigcap\{B: a \in B\,, B\ is\ closed\ in\ X\}$

(ii) D' denotes the set of all derived sets containing D in X

(iii) If E is a subset of the ideal space (X, τ, J), then E is g −open if $X - E$ is g −closed.

(iv) The α −closure of the $B \subseteq X$ (denoted by $Cl_\alpha(B)$)(Dunham, 1982) is the intersection of every α −closed set containing it.

(v) The α −interior of $E \subseteq X$ (denoted by $Int_\alpha(B)$) (Dunham, 1982), is the union of every α −open set contained in it.

(vi) The g −closure of $N \subseteq X$ (denoted by $Cl^*(N)$) (Császár, 2007) is the intersection of the g −closed sets containing it.

(vii) $\alpha O(X)$ denotes the family of α −open subsets of the ideal space X.

(viii) $\alpha C(X)$ denotes the family of α −closed subsets of X (Altawallbeh, 2020).

(ix) $GO(X)$ denotes the family of all generalized open subsets of X (Levine, 1963).

(x) $GC(X)$ denotes the family of all generalized closed subsets of X.

(xi) $O(X, x)$ is the set of all open subsets of X.

(xii) The function $f: X \rightarrow Y$ is α −continuous *if and only if* whenever $V \subseteq Y$ is α −open, $f^{-1}(V)$ is an α −open subset of Y.

2. Results and discussion

2.1. Countably α −compact ideal space

Numerous types of continuous functions have been developed over time, and continuity is a key idea in ideal topological spaces. In optimization issues, continuity is used to determine the function's maximum and minimum values in order to achieve a smooth change of state. Numerous applications of signal processing necessitate ongoing operations, such as the analysis and manipulation of signals in image and audio processing. Ideal topological spaces have a significant role in mathematics and quantum physics (Almuhur and Al-Labadi, 2022).

Definition 2.1.1: The ideal space X is a countably α −compact space if each countable set E of the open α −compact subsets N covering X has a finite subcover.

Theorem 2.1.2: Every α −compact ideal space is countably α −compact.

Proof. Let X be an α −compact ideal space, then each open cover of α −compact set has a finite subcover containing α −compact sets.

Remark 2.1.3: Every countably α −compact ideal space is countably compact, but the countably compact need not be countably α −compact ideal.

Example 2.1.4: If X is an ideal topological space such that $X = \{0\} \cup \{u_n: n \in \mathbb{N}\}$ and $\tau = \{\phi, \{0\}\}$, then X is countably compact but not an α −countably compact (Banakh and Bardyla, 2019).

Lemma 2.1.5: X is a countably α −compact ideal space *if and only if* $\tilde{F} = \{F_n: F_n\ is\ \alpha - closed, n \in \mathbb{N}\}$ is a countable family having the finite intersection property such that $\bigcap \tilde{F} \neq \emptyset$ if and only if $\bigcap_{n\in\mathbb{N}} F_n \neq \phi$ for each decreasing sequence $(F_n)_{n=1}^{\infty}$.

Proof. For the necessary part: Suppose that X is a countably α −compact ideal space, then for each family of α −open sets U_α such that $X = \bigcup_{\alpha\in\Lambda} U_\alpha$, $\exists finite\ U_{\alpha_0} \subseteq U_\alpha: X = \bigcup_{\alpha\in\Lambda} U_{\alpha_0}$.

For the sufficient part: If $\tilde{F} = \{F_n: F_n\ is\ \alpha - closed, n \in \mathbb{N}\}$ is a countable family having the finite intersection property such that $\bigcap \tilde{F} = \emptyset$ if and only if $\bigcap_{n\in\mathbb{N}} F_n \neq \phi$ for each decreasing sequence $(F_n)_{n=1}^{\infty}$, then we have a contradiction.

Theorem 2.1.6: The countable union of countably α −compact subspaces of an ideal space is countably α −compact.

Proof. Suppose that $\mathcal{A} = \{A_\alpha: \alpha \in \Gamma\}$ is a family of α −closed subspaces of the ideal space X. Also, let $\{U_\alpha: \alpha \in \Gamma\}$ be a countable α −open cover of the subspace Y such that $U_\alpha \subseteq Y$ is an α −open set $\forall \alpha \in \Gamma$ and $Y = \bigcup_{\alpha\in\Gamma} U_\alpha$.

Since Y is closed, $\exists\ \Gamma_\circ = \{\alpha_1, \alpha_2, \dots, \alpha_n\} \subseteq \Gamma$ such that $\bigcup_{\alpha\in\Gamma_\circ}\{U_{\alpha_n}: n \in \mathbb{N}\}$ is a countable subcover of the countably α −compact subspace Y. Hence, $(X - A_\alpha) \cap U_\beta$ covers X, $\forall \alpha \in \Gamma$, and $X = \bigcup_{n\in\mathbb{N}}(X - A_{\alpha_n}) \cup U_{\alpha_n}$ is a countable subcover of X. Thus, X is a countably α-compact ideal space.

Corollary 2.1.7: If Y is a closed subspace of X, then whenever F is an α −closed subset of X, F is an α −closed subset of Y.

Proof. Since X is an ideal space, $Y \subseteq X$ is closed, and $F \subseteq Y$, then F is an α −closed subset.

Theorem 2.1.8: Every locally finite family of non-empty α −open subsets of X is finite if *and only if* X is countably α −compact.

Proof. For the sufficient part, if X is a countably α −compact ideal space and some of its locally finite families of non-empty subsets are infinite, then $\exists\{F_n: F_n \text{ is } \alpha-\text{closed}, n \in \mathbb{N}\}$ such that $A_n = \bigcup_{n\in\mathbb{N}} F_n$ is decreasing, thus using Lemma 2.5, $\bigcap_{n\in\mathbb{N}} F_n$ is non-empty. Consequently, X is not a countably α −compact space, and this contradicts the assumption.

Corollary 2.1.9: If $\tilde{X} = \{(X_\alpha, \tau_\alpha, J_\alpha): \alpha \in \Lambda\}$ and Λ is finite, then the sum $\bigoplus_{\alpha\in\Lambda}(X_\alpha, \tau_\alpha, J_\alpha)$ is countably α −compact *if and only if* X_α is countably α −compact $\alpha \in \Lambda$

Proof. For the necessary part, if $\bigoplus_{\alpha\in\Lambda}(X_\alpha, \tau_\alpha, J_\alpha)$ is countably α −compact, for each $X_\alpha \neq \phi$, then X_α is α −compact $\alpha \in \Lambda$ and Λ is finite. The sufficient part is obvious.

2.2. α −continuous modulo (J, I)

Definition 2.2.1:

(i) If X and Y are two spaces and the function $f: X \to Y$ is α −continuous modulo (J, I), then $f^{-1}(W)$ is an α −open subset of X for all α −open subset W of Y.

(ii) A function $f: X \to Y$ is α −open modulo (J, I) if for every α −open (resp. α −closed) subset W of X, $f(W)$ is an α −open (resp. α −closed) subset of Y.

Lemma 2.2.2: Given two spaces, X and Y, and f is a function $f: X \to Y$, then:

(i) If f is α −continuous modulo (J, I), then f is α −continuous.

(ii) If f is a g −continuous function modulo (J, I), then f is α −continuous.

(iii) The function f is α −open *if and only if* for every neighborhood U of the point $a \in X$, $\exists V$ an α −open subset of Y containing $f(a)$ such that $V \subset f(U)$ (Altawallbeh, 2020)

(iv) f is α −compact if and only if for each $a \in X$ and W α −open (resp. α −closed) subset of Y containing $f(a)$, $\exists W'$ an α −open (resp. α −closed) subset of X: $f(W') \subset Y$.

Theorem 2.2.3: For the given spaces, (X, τ, J), (Y, σ, I) and (Z, η, K), and the functions $f: (X, \tau, J) \to (Y, \sigma, I)$ and $g: (Y, \sigma, I) \to (Z, \eta, K)$ where f is an α −continuous modulo (J, I), then $g \circ f$ is continuous modulo (I, K) but not α −continuous.

Theorem 2.2.4: If X and Y are α −compact spaces and the function $h: X \to Y$ is α −continuous, then the following are equivalent:

(i) h is bounded.

(ii) Countably α −compactness is an invariant property.

(iii) Every countably α −compact subspace (Z, η, I) of $(\mathbb{R}, \tau, J)$ is α −compact.

(iv) The class of countably α −compact spaces is perfect.

Corollary 2.2.5: The graph of the α −continuous function $g: (X, \tau, J) \to (Y, \sigma, I)$ modulo (J, I) is α −closed subset of $(X \times Y, \tau \times \sigma, J \times I)$.

Proof. Since g is an α −continuous function modulo (J, I), then g is an α −closed subset of $(X \times Y, \tau \times \sigma, J \times I)$.

Theorem 2.2.6: If X and Y are α −compact spaces, then $(X \times Y, \tau \times \sigma, I \times J)$ is a countably α −compact space but not α −compact.

Proof. By 2.2.5

Definition 2.2.7:

(i) The space X is called α −regular if $\forall a \in X$ and the α −closed subset F of X such that $a \notin F$, there exist two disjoint α −open subsets U_1 and U_2 of X such that $a \in U_1$ and F is dense in U_2 .

(ii) The space X is called completely α −regular if the points of X can be separated by α −continuous functions modulo J from α −closed subsets.

Theorem 2.2.8: Every α −closed subspace of the α −countably compact space X is α −compact.

Proof. If X is an α −compact space, then each cover of the subspace Y of X consisting of α −open sets has a finite subcover of α −open sets. Hence, Y is α −compact.

Corollary 2.2.9: If X is an α −countably compact space, and Y is a Tychonoff space, where $h: X \to Y$ is an α −continuous function modulo (J, I) and $\{A_n\}_{n\in\mathbb{N}}$ is a decreasing sequence of α −closed subsets of X, then $h(\bigcap_{n\in\mathbb{N}} A_n) = \bigcap_{n\in\mathbb{N}} h(A_n)$.

Proof. Since h is an α −continuous function modulo (J, I), then the result holds.

Theorem 2.2.10: If (X, τ, J) is a countably α −compact space, then:

(i) $|X| < e^{X^X}$

(ii) $|X| < c$, provided that X is first countable.

(iii) If $|X| < \aleph_0$, then X is the α −continuous image of the α −closed subspace of $D^{\aleph_0}$.

Corollary 2.2.11: The Cartesian product $(\prod_{\alpha\in\Lambda} X_\alpha, \prod_{\alpha\in\Lambda} \tau_\alpha, \prod_{\alpha\in\Lambda} J_\alpha)$ is countably α −compact *if and only if* $X_{\alpha\in\Lambda}$ is countably α −compact, $\forall\alpha \in \Lambda$.

Proof. Let $\prod_{\alpha\in\Lambda} X_\alpha$ be a countably α −compact space. Then the projection function $p_\alpha: X \to X_\alpha$ modulo $\prod_{\alpha\in\Lambda} J_\alpha$ is an α −continuous onto function.

Conversely, if $\{X_\alpha: \alpha \in \Lambda\}$ is a family of countably α −compact spaces, then $\prod_{\alpha\in\Lambda} X_\alpha$ is Hausdorff.

Let $\mathcal{A}=\{F_\alpha: \alpha \in \Lambda\}$ be a family of α −closed subsets of X_α, where X_α has the finite intersection property, then $\mathcal{A}$ is contained in the maximal family $\mathcal{F}$ which has the finite intersection property.

We claim that $\bigcap_{n\in\mathbb{N}} \mathcal{A}_n \neq \phi$, but $\mathcal{F}$ is maximal, so $\bigcap_{n\in\mathbb{N}} \mathcal{A}_n \neq \phi$. Let W be an α −open subset of X, then $W \cap \mathcal{A}_n \neq \phi, \forall n \in \mathbb{N}$.

Corollary 2.2.12: If $f: \prod_{\alpha\in\Lambda} X_\alpha \to \mathbb{R}$, is an α −continuous real-valued function modulo $\prod_{\alpha\in\Lambda} J_\alpha$ in the Cartesian product space $(\prod_{\alpha\in\Lambda} X_\alpha, \prod_{\alpha\in\Lambda} \tau, \prod_{\alpha\in\Lambda} J_\alpha)$ and $\{X_\alpha: \alpha \in \Lambda\}$ is a family of Hausdorff α −compact spaces that has a dense subset D, where $D = \bigcup_{\alpha\in\Lambda} A_\alpha$ for some α −compact subset A_α. Then X_α depends on countably many coordinates.

Proof. Let $\{X_{\alpha_n}: n \in \mathbb{N}\}$ be a non-increasing sequence of α −compact subsets of X_α, then $\bigcup_{n\in\mathbb{N}} X_{\alpha_n}$ is dense in X_α, by Corollary 2.20 and Lemma in Tyagi and Chauhan (2016).

Theorem 2.2.13: If (M, τ, J) is a subspace of the space (X, τ, J), and $h: (M, \tau, J) \to (Y, \sigma, I)$ is an α −continuous function, where (Y, σ, I) is an α −compact space. Then h has an extension $H: (X, \tau, J) \to (Y, \sigma, I)$ *if and only if* for all α −closed subsets $h(M_1)$ and $h(M_2)$ of Y, $f^{-1}(M_1) \cap f^{-1}(M_2) = \phi$ in X. Then, X is countably α-pseudocompact if and only if every α-continuous real-valued function defined on (X, τ, J) is bounded.

Proof. ⟹) Suppose that $h: (M, \tau, J) \to (Y, \sigma, I)$ has an extension $H: (X, \tau, J) \to (Y, \sigma, I)$, if $B_i \subset \overline{B_i} \subset Y\ \forall i = 1,2,$ and B_1 and B_2 are disjoint, then $H^{-1}(B_i) \subset \overline{H^{-1}(B_i)}$ and $H^{-1}(B_1) \cap H^{-1}(B_2) = \emptyset$. Hence, $h^{-1}(B_1) \cap h^{-1}(B_2) \subset H^{-1}(B_1) \cap H^{-1}(B_2) = \emptyset$.

For the insufficient part, suppose that $\beta(x)$ is the family of neighbourhoods of $x\ \forall x \in X$, and let $\hbar(x) = \{\overline{h(C \cap U_i)} : U_i \in \beta(x)\}_{i\in\mathbb{N}}$ where C is an α −compact subset of Y.

Now, $\overline{h(C \cap U_1 \cap U_2 \cap \ldots \cap U_n)} \subset \overline{h(C \cap U_1)} \cap \overline{h(C \cap U_2)} \cap \ldots \cap \overline{h(C \cap U_n)}$.

Thus, $\hbar(x)$ has the finite intersection property and so $\cap \hbar(x) \neq \phi$. So, h is bounded (Al-Omari, 2019).

Definition 2.2.14: If X is a Tychonoff space, then X is countably α −pseudocompact, if any α −continuous real-valued function defined on X is bounded.

Theorem 2.2.15: If X is a Tychonoff space, then the following are equivalent:

(i) X is countably α −pseudocompact.
(ii) Every locally finite family of α −open subsets of X is finite.
(iii) Every locally finite open cover of X consisting of α −open subsets of X is finite.

Proof. (i) → (ii) Let X be a countably α −pseudocompact space.

Let $\mathcal{V} = \{V_n: n \in \mathbb{N}\}$ be a locally finite family of α −open subsets of X.

Since X is a Tychonoff space, then there exists a real-valued α −continuous function $g_n: (X, \tau, J) \to (\mathbb{R}, \sigma, I)$ modulo (J, I) such that $g_n(x_n) = n$, for some $x_n \in V_n$ and $g_n(X - V_n) \subset \{0\}$.

Now, $\{V_n: n \in \mathbb{N}\}$ is a locally finite family, then $g(x) = \sum_{n\in\mathbb{N}} g_n(x)$ is a real-valued α −continuous function that is not bounded. Thus, X is not an α −pseudocompact space.

Corollary 2.2.16: If X is a Tychonoff space, then the following are equivalent:

(i) X is a countably α −pseudocompact space.
(ii) If $\mathcal{V} = \{V_n: n \in \mathbb{N}\}$is a family of α −open subsets of X with the finite intersection property, then $\bigcap_{n\in\mathbb{N}} V_n \neq \phi$.

Proof. For (i) → (ii). Assume that X is a countably α −pseudocompact set and that $\{V_n: n \in \mathbb{N}\}$ is a decreasing sequence of α −open subsets of X, then $\mathcal{V} = \{V_n: n \in \mathbb{N}\}$ is not a locally finite family.

Hence, for some $a \in X$, the neighborhood of a meets infinitely many sets V_n , $\forall n \in \mathbb{N}$. Therefore, $\bigcap_{n\in\mathbb{N}} V_n \neq \phi$.

For (ii) → (i) it is obvious.

Lemma 2.2.17: If $h: X \to Y$ is a surjective α −continuous function modulo (J, I), X is countably α −pseudocompact and Y is a Tychonoff space, then Y is a countably α −pseudocompact space.

Theorem 2.2.18: The Cartesian product of a countably α −pseudocompact space and an α −compact space is countably α −pseudocompact.

Theorem 2.2.19: The countably α −pseudocompactness property is not hereditary with respect to α −closed subsets.

Corollary 2.2.20: The countably α −pseudocompactness property is not finitely multiplicative.

Corollary 2.2.21: If X is a Tychonoff space, Y is a countably α −pseudocompact space, and the perfect function $f: X \to Y$ is α −continuous modulo (J, I), then X is countably α −psuedocompact.

3. Conclusions

Ideal topological spaces that are countably α −compact were first proposed by Almuhur and Bin-Asfour (2022). The α −continuous function modulo (J, I) that maps the ideal space (X, τ, J) that is countably α −compact to the ideal space (Y, σ, I) is an α −closed subset of the Cartesian product $(X \times Y, \tau \times \sigma, J \times I)$.

The α −continuous image of the closed subspace of the cube $D^{\aleph_0}$ is the countably α −compact ideal space (X, τ, J) that weighs more than or equal to $\aleph_0$. Furthermore, subject to certain restrictions, the α −continuous function $f: (X, \tau, J) \rightarrow (Y, \sigma, I)$ modulo (J, I) where Y is countably α −compact can be extended over its domain.

It is demonstrated that the countably α −pseudocompactness is neither finitely multiplicative nor hereditary with respect to α −closed sets. The α −pseudocompactness is defined in the ideal topological space (X, τ, J). Additionally, it is discovered that the Tychonoff space is countably α −pseudocompact and that the α −continuous function modulo ideals mapping it to a Tychonoff ideal space is perfect.

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Compliance with ethical standards

Conflict of interest

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